PROOFS BY CONTRADICTION REVEAL 445 YEAR OLD MATH SHOCK!

(i) DEFINITION OF POSITIVE INTEGRAL MULTIPLICATION (since 1570) *ab = a added to itself b times*

What do math professors assert? *"...the product of two quantities is calculated, usually written a x b, a.b, or ab. To multiply a by integral b is to add a to itself b times"* [1] *"In arithmetic, multiplication of one number, a, by another, b, consists of adding a to itself b times. This kind of multiplication is commutative, that is, $a \times b = b \times a$."* [2] *"Does seven times three, mean seven added to itself three times, or three added to itself seven times?"* [3] *"What is 17 x 12? By definition, this is 12 added to itself 17 times."* [4]

THEOREM I *Multiplication distributes with addition: $(a + b) \times n = a \times n + b \times n$*

Via (i) Let $(a + b) \times n = (a + b)$ added to itself n times.

CASE 1 Let $n = 1$

$(a + b) \times 1 = (a + b)$ added to itself 1 time.

$(a + b)$ added to itself 1 time $= (a + b) + (a + b)$ That is, one addition of $a + b$ to itself.

$(a + b) + (a + b) = (a + b) \times 2$, yet...

$(a + b) \times 1 \neq (a + b) \times 2$

CASE 2 Let n = 1

$(a + b) \times 1 = a \times 1 + b \times 1$

Via (i) $a \times 1 = a$ added to itself 1 time $= a + a$ (i.e. one addition of a to itself)

Via (i) $b \times 1 = b$ added to itself 1 time $= b + b$ (i.e. one addition of b to itself)

$(a + b)$ added to itself 1 time $= (a + a) + (b + b) = 2a + 2b = (a + b) \times 2$, yet...

$(a + b) \times 1 \neq (a + b) \times 2$

THEOREM I FAILS *Multiplication, as defined, does NOT distribute over addition.*

THEOREM II *Multiplication is commutative a x b = b x a*

CASE 3 Let $a = 1$ and $b = 1$

Via (i) $a \times b = 1$ added to itself 1 time $= 1 + 1$ (i.e. one addition of 1 to itself)

Via (i) $b \times a = 1$ added to itself 1 time $= 1 + 1$ (i.e. one addition of 1 to itself)

THEOREM II HOLDS WHEN a = b = 1

CASE 4 Let $a = 2$ and $b = 2$

Via (i) $a \times b = 2$ added to itself 2 times $= 2 + 2 + 2$ (i.e. two added to itself twice)

Via (i) $b \times a = 2$ added to itself 2 times $= 2 + 2 + 2$ (i.e. two added to itself twice)

THEOREM II HOLDS WHEN $a = b = 2$ and (via induction) when $a = b = n$

CASE 5 Let $a = 1$ and $b = 2$

Via (i) $a \times b = 1$ added to itself 2 time $= 1 + 1 + 1$ (i.e. one added to itself twice)

Via (i) $b \times a = 2$ added to itself 1 time $= 2 + 2$ (i.e. two added to itself once). yet...

THEOREM II FAILS WHEN $a < b$ as $1 + 1 + 1 \neq 2 + 2$

CASE 6 Let $a = 2$ and $b = 1$

Via (i) $a \times b = 2$ added to itself 1 time $= 2 + 2$ (i.e. two added to itself once)

Via (i) $b \times a = 1$ added to itself 2 time $= 1 + 1 + 1$ (i.e. one added to itself twice), yet...

THEOREM II FAILS WHEN $a > b$ as $2 + 2 \neq 1 + 1 + 1$

Multiplication, as defined for 445 years, neither commutes, nor computes!

Jonathan Crabtree

THEOREM II FAILS *Multiplication, as defined, does NOT commute unless a = b.*

"Well, what if we multiply two fractions? Say, 1/3 x 1/2? Uh oh. This is now a problem since it doesn't make sense to think of adding 1/2 to itself 1/3 of a time!" [5] "How, for example, does one add 5/8 to itself 3/4 times, d to itself π times, or –2 to itself –3 times?" [6] **People please note:** Adding ***a*** to itself ***b*** times does not equal ***ab***. In ***ab***, if ***b*** = 1, then ***a*** × 1 or ***a***, cannot be ***a*** added to itself one time. The is because a added to itself one time equals ***2****a*, not ***a***. In ***ab***, if ***b*** = 0 then ***a*** × 0 or 0, is not ***a*** added to itself zero times, because ***a*** added to itself zero times is ***a***, not 0.

Discover how the world was pranked by a London haberdasher for 445 years! *(More to come!) Connect to Jonathan Crabtree at LinkedIn to be the first to find out how abstract arithmetic REALLY works!)*

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1 Dr. Ephraim Borowski & Dr. Jonathan Borwein, *Collins Dictionary of Mathematics*, P.129, Harper Collins, 2012. Also online at http://www.collinsdictionary.com/dictionary/english/multiplication

2 Dr. John Daintith, *The Facts On File Dictionary of Mathematics*, P. 143, Market House Books, 2005.

3 Dr. Steven Strogatz, *The Joy of X: A guided tour of math, from one to infinity*, P.23, Houghton Mifflin Harcourt, 2012.

4 Dr. Hung-Hsi Wu, *Understanding Numbers in Elementary School Mathematics*, P. 58, American Mathematical Society, 2011.

5 Dr, Jason Marshall, www.quickanddirtytips.com/education/math/is-multiplication-repeated-addition

6 Dr. Brent Davis, *The Best Writing on Mathematics 2012*, P136, Princeton University Press, 2014.